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Phase Transitions at Preheating

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Abstract

Symmetry restoration processes during the non-equilibrium stage of "preheating" after inflation is studied. It is shown that symmetry restoration is very efficient when the majority of created particles are concentrated at energies much smaller than the temperature T in equilibrium. The strength of symmetry restoration measured in terms of the equivalent temperature can exceed T by many orders of magnitude. In some models the effect can be equivalent to that if the temperature of instant reheating would be close to the Planck scale. This can have an important impact on GUT and axion models.

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In accordance to modern cosmology the outcome of the earliest stages of the Universe evolution, which predefines its modern appearance, is determined by fine details of the dynamics of a particular scalar field, which is called inflaton. For a review of inflationary models see Ref. [1]. The chaotic inflation [2] is a typical model which has essential common features. In this model the inflationary stage itself persists till slowly decreasing inflaton field, $\phi(t)$, is larger than the Plank scale, $M_{\rm Pl} \approx 10^{19}$ GeV. During this stage the Universe expands exponentially and the room for the future matter is created. This stage ends when the inflaton field reaches $\phi \sim M_{\rm Pl}$ and then the field starts to oscillate coherently. Coherently oscillating field can be considered as a collection of unstable inflaton quanta at rest, so it decays to all particles it has coupling with, and the matter is created.

With the assumption of "instant" reheating the products of inflaton decay would thermalize on time scale negligible compared to the rate of the Universe expansion, so the temperature after reheating would be $T_{\rm eq}^4 = \rho_\phi/g_*$, where numerical factor g_* includes total number of degrees of freedom and is large, $g_* \gtrsim 10^2$. Here ρ_ϕ is the initial energy density stored in inflaton oscillations $\rho_\phi \sim \lambda\phi^4/4 \sim \lambda M_{\rm Pl}^4$. It is relatively low since the inflaton self-coupling constant has to be very small $\lambda \sim 10^{-13}$ for the induced density perturbations to satisfy observational constraints. In reality, the reheating temperature will be even much smaller since the reheating is not instant and while the particles thermalize, the Universe expands and cools.

The magnitude of the reheating temperature after inflation is considered to be important as, for example, this will determine whether or not certain scenarios of baryogenesis in Grand Unified Theories (GUT) of strong and elecktroweak interactions will be successful which requires the reheating to be up to the GUT scale $M_X \sim 10^{16}$ GeV. Another important issue is the occurrence of phase transitions. If the reheating temperature is larger than the Grand Unification symmetry breaking scale, then the corresponding symmetry will be restored (for reviews of phase transitions in GUT, see, e.g. [3,1]). The subsequent cooling will be accompanied by a symmetry breaking phase transition which will proceed in different horizon volumes independently, resulting in creation of topological defects: domain walls,

strings and magnetic monopoles. This means that the problem of monopoles [4] and domain walls [5] will be resurrected, ruling out the corresponding models.

The question of symmetry restoration is interesting not only in connection to topological defects. Another important aspect is whether the Peccei-Quinn (PQ) symmetry [6] is restored after inflation, or not. PQ symmetry was introduced to explain the apparent smallness of CP-violation in QCD. Pseudo-Nambu–Goldstone boson resulting from the spontaneous breaking of this symmetry, the invisible axion, is among the best motivated candidates for cosmic dark matter. The combination of cosmological and astrophysical considerations restrict the relevant symmetry breaking scale, or axion decay constant f_a, to be in the narrow window $10^{10}\,\mathrm{GeV} \leq f_a \leq 10^{12}\,\mathrm{GeV}$ (for a review see [7], note that the upper bound on f_a does not apply in certain inflationary models [8]). If the PQ symmetry is restored after inflation, then the axion field will not be constant throughout the Universe, but will have independent values in different horizons. These fluctuations in the axion field are transformed into density fluctuations of order unity at the crucial epoch when the axion mass switches on at $T \approx 1$ GeV, leading to the existence of very dense axion miniclusters [9,10], which may be observable [11]. This also shifts the main source of axions from a coherent misalignment angle to decaying axion strings [12].

Traditionally, the answer to all of the above questions was associated with the value of the reheating temperature after inflation. The purpose of the present Letter is to show that the reheating temperature is actually irrelevant here and all processes of interest are even more efficient while the system is still out of equilibrium. This is more or less apparent for the baryogenesis since one of the necessary conditions for baryon number generation is the absence of equilibrium. We shall show that symmetry restoration is more efficient too in a non-equilibrium state generated at the final stage of inflation.

Recently it was realized that in certain cases the decay of the inflaton field can be a very fast process [13–15] (see also [16]), owing to the possibility of stimulated decays. This is also called the parametric resonance, for the general theory of it see, e.g. [17]. Sometimes the parametric resonance is considered as a very special type of decay of coherent field which

can not be described as a decay of particles at rest. This is an incorrect conclusion - one just has to include stimulated processes in addition to spontaneous [18] for the particles decay or annihilation. That is why parametric resonance exists only for Bosons in the final state. But even in Bose systems the parametric resonance is not necessarily always effective. For example, it was studied long ago for the decay of the axion field [19,18] with the negative conclusion that the expansion of the universe removes particles from the narrow resonance zone too quickly, blocking the entire process (to reach stimulated decays of axions bound in a gravitational well is not impossible in principle, but would require enormous densities of particles in this particular case [18,20]). This means, in particular, that if the inflaton potential and inflaton interactions are constructed in analogy with the axion potential, which is the case in the model of "natural" inflation [21], then the parametric resonance can be ineffective. However, as has been demonstrated in [13], a successful parametric resonance can occur in the case of, e.g., chaotic inflation [2].

In the case of successful resonance almost all energy stored in the form of coherent inflaton oscillations is transferred almost instantaneously to radiation, but the products of inflaton decay are still far from equilibrium. This intermediate stage was dubbed "preheating" in Ref. [13]. To simplify subsequent discussion let us first take the distribution function of created particles to be of the form

$$f(p) = A\delta(p_0 - E) \tag{1}$$

For the case of two particle decay E in this equation is equal to half the inflaton mass, for $2 \rightarrow 2$ annihilation $E = m_\phi$, for the processes $4 \rightarrow 2$ of self-annihilation we have $E = 2m_\phi$, etc. The main point which is crucial for the subsequent discussion is that E is typically smaller by many orders of magnitude than the temperature T of instant reheating. Equilibration will take time, meanwhile particular phase transitions can occur in the system when it is still far from the equilibrium and the distribution function is still given by Eq. (1). Moreover, we shall show that in this case the symmetry restoration is even much more efficient.

Before we proceed, let us specify the general field and particle contents of the system. First, there is classical inflaton field which we already introduced as ϕ. We denote as m_ϕ the effective inflaton mass at the Plank scale, which includes contribution from the self-interaction $m_\phi^2(\phi) = m_\phi(0)^2 + 3\lambda\phi^2$. For us it is only important that the overall value of m_ϕ is fixed to $m_\phi \sim 10^{13}$ GeV by the observed magnitude of density perturbations. Second, there are products of inflaton decay. We denote them as η and the mass of corresponding quanta is m_η. The possibility of the inflaton decay into η-quanta assumes that there is interaction of, say, the form $g\eta^2\phi^2/2$. In reality, there can be many channels for the inflaton decay and the final answer is the sum over all species. Since the true content is unknown, we shall not carry out this summation, but implicitly assume that it has to be done. Third, there is relevant order parameter, the classical field Φ. The η-particles couple to the order parameter, so that their mass depends upon it, $m_\eta = m_\eta(\Phi)$. The typical case is $m_\eta^2(\Phi) = m_\eta(0)^2 + \alpha\Phi^2$ with α being a product of coupling constant and some numerical factor which depends upon particular direction in the space of internal symmetries. In the case of simple one-component scalars this assumes interaction of the form $\alpha\eta^2\Phi^2/2$.

Note, that since we have to carry out the summation over all channels, there will be terms when η corresponds to quanta of the ϕ or Φ fields, and even when ϕ, η and Φ is one and the same field. In some models some particular terms can be negligible. The simplest possibility corresponds to the only one dominant term $\phi = \eta = \Phi$. While we do not exclude this possibility, in order to keep uniformity we shall keep separate notations for the separate aspects of one and the same entity. For example, we shall still denote by η the quanta of the ϕ field, etc.

In the vacuum state the symmetry is broken, which can be described at the tree level by the potential $V_0 = -\mu^2\Phi^2/2 + \lambda_\Phi\Phi^4/4$. This relates μ to the symmetry breaking scale via $\mu^2 = \lambda_\Phi\Phi^2$. At non-zero density of particles there are corrections to this potential. In particular, $m_\Phi^2(0) \equiv d^2V_0/d\Phi^2\,(\Phi = 0)$ became positive and the symmetry is restored. To calculate the modified potential we can make use of the fact that the effective potential is minus the pressure. This definition is physically transparent: if two phases can coexist,

the phase with the lower value of the effective potential will have higher pressure and the bubbles of this phase will eventually occupy the whole volume.

The pressure P for an arbitrary distribution function of η-particles can be found by using the formula

$$T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{p^0} f(p)\,, \tag{2}$$

where $p_0^2 = p^2 + m_\eta^2(\Phi)$ and for an isotropic medium we have $T^{ij} = \delta^{ij} P$. We omit the subscript η for the distribution function since η are the only particles we have.

While the field Φ evolves in the effective potential, the number of particles does not change on time scales of interest, so we shall calculate (2) assuming that

$$N = \int \frac{d^3p}{(2\pi)^3} f(p)\,, \tag{3}$$

is constant which fixes the normalization factor A in Eq. (1). The procedure is very simple and the result for the particle-dependent part of the effective potential is $V_1(\Phi) = -P = N\,[m_\eta^2(\Phi) - E^2]/3E$. Since for the distribution function given by Eq. (1) the energy density is simply $\rho = NE$, and since with the assumption of instant reheating the energy density would be unchanging, it is convenient to rewrite our result in the form

$$V_1(\Phi) = \frac{\rho}{3E^2} m_\eta^2(\Phi) - \frac{\rho}{3}\,. \tag{4}$$

At $m_\eta^2 = 0$ we recover the equation of state for massless particles, $P = \rho/3$. But ρ is a Φ-independent constant and is insignificant for us here. What we are interested in is the coefficient in front of $m_\eta^2(\Phi)$, which we denote B,

$$B = \frac{\rho}{3E^2}. \tag{5}$$

This coefficient is positive, so when added to the negative mass square of the field Φ in vacuum, leads to the restoration of the symmetry at large density of particles, $V''(0) = -\mu^2 + 2\alpha B$. The symmetry is restored at $B > \mu^2/2\alpha \sim (\lambda_\Phi/\alpha)\Phi^2$ (in the typical case of positive α; negative α is also possible in models with several scalar fields, see Ref. [3], which breaks the symmetry instead).

If we were to calculate the effective potential with an equilibrium thermal distribution we would obtain $B_{\rm eq} = T^2/24$, see Ref. [22]. Roughly, in Eq. (5), we would have in this case $\rho \sim T^4$ and $E \sim T$. We can generalize the expression for B as been given by the ratio of particle density to the mean energy of particles for a case when the width of the distribution is finite, but still it is smaller than E.

Now we can compare the effectiveness of the symmetry restoration at preheating to the instant reheating (actual reheating in the expanding Universe is even less effective). In both cases the energy density is the same and is equal to initial inflaton energy density, but at preheating $E \ll T$. The restoration of the symmetry is much more effective in the non-equilibrium state Eq. (1), its strength amplified by $B \sim (T/E)^2 B_{\rm eq}$.

Let us make an order of magnitude estimate for B in some possible inflationary models. The inflaton field strength at the end of inflation is of order $M_{\rm Pl}$, so that the energy density in inflaton oscillations is given by $\rho \sim m_\phi^2 M_{\rm Pl}^2$. Its magnitude is fixed by the magnitude of primordial density perturbations. To use Eq. (5) we only need the estimation of E. We can consider three cases here.

1) If $E \sim m_\phi$ we obtain $B \sim M_{\rm Pl}^2$, which is equivalent to a would-be reheating temperature up to the Plank scale. Note, however, that we had not calculated the numerical coefficient here, which can be rather small. One example when this regime can be valid corresponds to the self-annihilation of the inflaton field, i.e. $\eta = \phi$ dominated case. Decay corresponding to the $4 \to 2$ processes is unsuppressed in the expanding universe if the inflaton mass is dominated by the self-interaction, and $E \approx 2m_\phi(\phi)$ [13].

This effect will cause the inflaton field to "roll back" to some extent in the new inflationary scenario (the inflaton can not roll all the way back to the origin if $\phi \approx 0$ was the initial condition), which was observed in the detailed numerical simulations of Ref. [15].

2) Let us consider the axion case (or to this extent any model with sufficiently low value of symmetry breaking scale). The Peccei-Quinn symmetry breaking scale $f_a \lesssim 10^{12}$ GeV is comparable to the inflaton mass. Therefore the contribution to the mass of the η particles due to interaction with PQ field is smaller than the inflaton mass and no additional

kinematical constraints appear. However, the inflaton will decay into lowest zones where $E \sim m_\phi$ only if $g \sim \lambda$. This happens at $g \ll \lambda$ too, but then the dominant process is self-annihilation of the inflaton to its own quanta. For the case $g \gtrsim \lambda$ the mean energy of created particles was found in Ref. [13] to be given by $\bar{E}^2 \sim g^{1/2} m_\phi M_{\rm Pl}$ and we find $B \sim g^{-1/2} m_\phi M_{\rm Pl} \sim (\lambda/g)^{1/2} M_{\rm Pl}^2$. Nevertheless, this might not reduce the coefficient B significantly since in a models without special symmetry cancellations the constant g can not be very large, otherwise loop corrections will induce unacceptably large self-coupling for the inflaton. In such models we have $g \lesssim \sqrt{\lambda}$. Moreover, even assuming $g \lesssim 1$, for the effective equivalent temperature which would result to the same strength of the symmetry restoration we find $T_{\rm eff} \gtrsim (\lambda)^{1/4} M_{\rm Pl} \sim 10^{-3} M_{\rm Pl}$.

The Peccei-Quinn symmetry is restored if $B \gtrsim (\lambda_\Phi/\alpha) f_a^2 \sim 10^{-14} (\lambda_\Phi/\alpha) M_{\rm Pl}^2$. We see that the PQ symmetry is restored at preheating if $\alpha > 10^{-7} \lambda_\Phi \sqrt{g}$, which is a rather weak condition. Note also that in chaotic inflationary model the PQ symmetry can be restored anyway already during inflationary stage, just due to possible direct coupling of inflaton and PQ fields [23]. We conclude that the "thermal" scenario for the axion evolution has strong support.

3) Now let us consider the case of GUT which has large magnitude of the symmetry breaking scale $\Phi \sim 10^{16}$ GeV. Since E can not be smaller than m_η, the parameter B is suppressed for η particles which have large coupling to Φ, and which correspondingly have large masses M_X. However, at fixed coupling g the decays to instability zones with large energy are suppressed, and we possibly can neglect those channels even if the value of g is large. In Ref. [13] it was shown that the creation rate of particles does not depend much upon g near the surface of zero energy of created particles. Consequently the inflaton will preferably decay to particles which have low value of α. Since the particle content of the theory is large, we can expect that particles satisfying the condition $m_\eta^2 \sim \alpha \Phi^2 < g^{1/2} m_\phi M_{\rm Pl}^2$ when there is no additional suppression, can be found. For the case of GUT's this translates to the condition $\alpha \lesssim \sqrt{g}$. Then, the GUT symmetry is restored if $B \gtrsim 10^{-6} (\lambda_\Phi/\alpha) M_{\rm Pl}^2$.

Using $B \sim (\lambda/g)^{1/2} M_{\rm Pl}^2$ we find the condition $\alpha > \lambda_\Phi \sqrt{g}$. Combining both conditions we see that η particles which will satisfy $\lambda_\Phi \sqrt{g} < \alpha < \sqrt{g}$ (if exist) can restore the GUT symmetry.

If parametric resonance is still efficient for particles which does not satisfy the condition $\alpha < \sqrt{g}$, we obtain $B \sim m_\phi^2 M_{\rm Pl}^2/\alpha\Phi^2 \sim 10^{-7} M_{\rm Pl}^2/\alpha$. Symmetry restoration can be possible if $\lambda_\Phi < 10^{-1}$.

As final remarks let us discuss the applicability of our approach to the effective potential based on Eq. (2). In the usual frameworks developed for equilibrium in Refs. [22] this would correspond to one-loop approximation to the effective potential. No significant further approximations are made despite the situation is non-equilibrium. As compared to the equilibrium, higher loops can be important in the present case, however, and this issue deserves separate study. Note also, that the effective potential is not a well defined notion in itself, but the effective potential can be defined for the case of special field configurations, and this is sufficient for our purposes. The effective potential is equal to minus pressure if one restricts to critical bubbles and it is equal to energy density if one considers homogeneous field configurations only. Equations of motion for arbitrary field configurations can also be easily found in the present approach. It is sufficient to use continuity of the stress-energy tensor, $\partial_\nu T^{\mu\nu} = 0$, with $T^{\mu\nu}$ being the sum of Eq. (2) and the stress-energy of the free field Φ. With the use of the Liuville theorem for $f(p,x)$ this continuity condition reduces to the equation for the field Φ,

$$\Box\Phi + \frac{dV_0}{d\Phi} + \frac{dm_\eta^2}{d\Phi} \int \frac{d^3p}{(2\pi)^3} \frac{f}{2p^0} = 0\,. \tag{6}$$

One of the advantages of our approach to the calculation of the effective potential being based on Eq. (2) (or Eq. (6)) is that it allows us to find how the coefficient B changes when the distribution function evolves according to the kinetic equation [24], approaching an equilibrium.

In the present discussion we have neglected numerical factors like those which arise due to expansion of the Universe. Those will decrease value of B somewhat. On the other hand the usually employed description of stimulated decays based on the Mathieu equation takes

into account only the processes of the form $n \to 2$. However, at large phase-space density of particles when $gf(p) > 1$ the processes $n \to m$ with $m > 2$ start to dominate. This might reduce the evaluation of $\bar{E}$ of Ref. [13] and increase the value of B.

We conclude that physical processes at preheating are very important, especially with regard to problems of symmetry restoration, and deserve detailed study.

When the first version of this work was finished I became aware of Ref. [25] where similar conclusions were reached. I am grateful to the authors of Ref. [25] for correcting some of the statements contained in the original version of my paper.

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REFERENCES

[1] A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood, Chur, Switzerland, 1990).

[2] A. D. Linde, Phys. Lett. **108B** (1982) 389; A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48** (1982) 1220.

[3] V. A. Kuzmin, M. E. Shaposhnikov and I. I. Tkachev, Sov. Sci. Rev. A Phys. Rev., **8** (1987) 71-146.

[4] Ya. B. Zel'dovich and M. Yu. Khlopov, Phys. Lett. **79B** (1978) 239.

[5] Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun', Sov. Phys. JETP **40** (1974) 1 [ZhETF **67** (1974) 3].

[6] R. D. Peccei and H. Quinn, Phys. Rev. Lett. **38** (1977) 1440 .

[7] M. S. Turner, Phys. Rep., **C197** (1990) 67; G. G. Raffelt, Phys. Rep **C198** (1990) 1.

[8] A. D. Linde, Phis. Lett. **B259** (1991) 38.

[9] C. J. Hogan and M. J. Rees, Phys. Lett. **B205** (1988) 228 .

[10] E. Kolb and I. I. Tkachev, Phys. Rev. Lett. **71** (1993) 3051 ; Phys. Rev. D **49** (1994) 5040.

[11] E. Kolb and I. I. Tkachev, *Femtolensing and Picolensing by Axion Miniclusters*, preprint FERMILAB-Pub-95/-309A, OSU-TA-20/95, astro-ph/9510043, to appear in Ap. J. Lett.

[12] R. Davis, Phys. Lett. **B180** (1986) 225 ; D. Harari and P. Sikivie, Phys. Lett. **195** (1987) 361 ; R. A. Battye and E. P. S. Shellard, Phys. Rev. Lett. **73** (1994) 2954.

[13] L. Kofman, A. Linde, and A. A. Starobinsky, Phys. Rev. Lett. **73** (1994) 3195.

[14] Y. Shtanov, J. Traschen, and R. Brandenberger, Phys. Rev. D **51** (1995) 5438.

[15] D. Boyanovsky, H. J. de Vega, R. Holman, D. S. Lee, and A. Singh, Phys. Rev. D **51** (1995) 4419; PITT 17/95 (1995), hep-ph/9507414.

[16] A. D. Dolgov and D. P. Kirilova, Sov. Nucl. Phys. **51** (1990) 172; J. Traschen and R. Brandenberger, Phys. Rev. D **42** (1990) 2491.

[17] A. A. Grib, S. G. Mamaev, and V. M. Mostepanenko, *Quantum Effects in Strong External Fields*, [in Russian], Atomic Energy Press, Moscow (1980).

[18] I. I. Tkachev, Sov. Astron. Lett. **12** (1986) 305 ; I. I. Tkachev, Phys. Lett. **B191** (1987) 41 .

[19] J. Preskill, M. B. Wise, and F. Wilczek, Phys. Lett. **120B** (1983) 127; L. F. Abbott and P. Sikivie, Phys. Lett. **120B** (1983) 133.

[20] T. W. Kephart and T. J. Weiler, Phys. Rev. **D52** (1995) 3226.

[21] K. Freese, J. A. Frieman, and A. V. Olinto, Phys. Rev. Lett. **65** (1990) 3233.

[22] D. A. Kirzhnits, JETP Lett. **15** (1972) 529; D. A. Kirzhnits and A. D. Linde, Phys. Lett. **42B** (1972), Sov. Phys. JETP **40** (1974) 628, Ann. Phys. **101** (1976) 195; S. Weinberg, Phys. Rev. **D9** (1974) 3320; L. Dolan and R. Jackiw, Phys. Rev. **D9** (1974) 3357.

[23] L. A. Kofman and A. D. Linde, Nucl. Phys. **B282** (1987) 555.

[24] D. V. Semikoz and I. I. Tkachev, in preparation.

[25] L. Kofman, A. Linde, and A. A. Starobinsky, hep-th/9510119, Phys. Rev. Lett. **76** (1996) 1011.